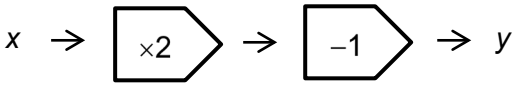
OCR 06 Algebra (Higher)

1. Simplify $5x^2 \times 4x^{-5}$.

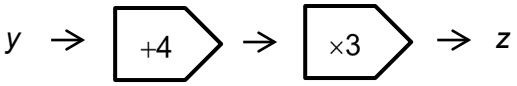
2. Simplify $8y^{\frac{1}{2}} \div 2y^{\frac{3}{2}}$.

3. A function is given by $y = 5x - 3$. Write an expression for the inverse of this function.

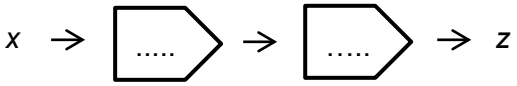
4. A value, x, is input into this function.



The output, y, is then input into this function.



Complete the function below.



5. Find the value of x in the following.

$$3(x-4) = 2(8-2x).$$

6. Write $x^2 + 8x + 5$ in the form $(x+a)^2 + b$.

7. Simplify $\dfrac{x^2 - 3x - 18}{x^2 + 4x + 3}$.

8. Rearrange the formula $y = \dfrac{x+3}{x-2}$ to make x the subject.

9. Factorise $3x^2 + 7x - 20$.

10. Find the formula for the nth term of the quadratic sequence 5, 12, 25, 44, 69, ….

11. Kasim adds together two fractions. His working is shown below.

$$\frac{2}{x+1} + \frac{1}{x-2} = \frac{2+1}{x+1+x-2} = \frac{3}{2x-1}$$

Identify Kasim's error and calculate the correct answer.

12. By completing the square, show that the solutions to the equation $x^2 - 4x - 7 = 0$ are $x = 2 - \sqrt{11}$ and $x = 2 + \sqrt{11}$.

13. Prove algebraically that the difference between the squares of two consecutive **odd** integers is even.

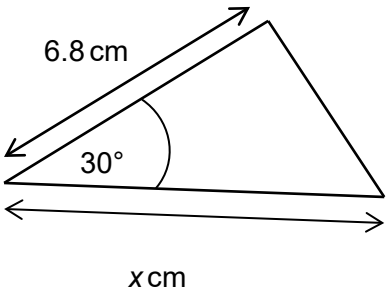
14. A formula for the *n*th term of the sequence 6, 11, 20, 33,… is $2n^2 - n + 5$. Write down a formula for the *n*th term of the sequence 10, 15, 24, 37,... and use it to show that 240 is a term of this sequence.

15. The equation $x^3 + x - 85 = 0$ has a solution between 4 and 5.
Show that this solution is 4.3 to one decimal place.

16. List the integer values that satisfy both of the inequalities $2y - 3 > -5$ and $3y + 1 < 13$.

17. A Fibonacci type sequence is given by $x_{n+2} = x_n + x_{n+1}$.
$x_1 = a$ and $x_2 = b$. Write expressions for the third and fifth terms. If the third term is 11 and the fifth term is 39, find the values of *a* and *b*.

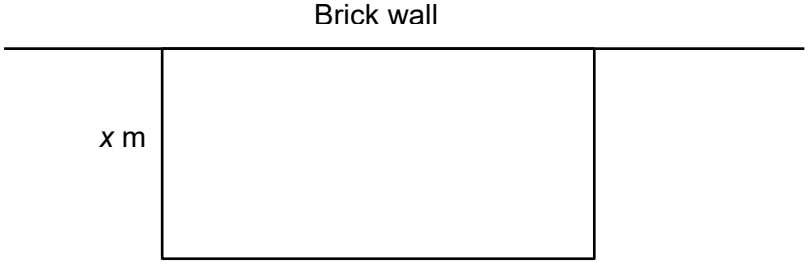
18. The area of the triangle below is 14.28 cm^2. Find the value of *x*.



Not to scale

19. A straight line has equation $y = 2x - 1$. A circle has equation $x^2 + y^2 = 9$. Find the coordinates of the points of intersection of the circle and the straight line. Give your answers correct to 3 significant figures.

20. Sarah wants to make a rectangular pen for her rabbits. One side of the pen will be a brick wall and she has 16 m of wire fencing for the other three sides. Find possible values for *x* if the area of the pen is at least 24 m^2.



Answers

1. $20x^{-3}$

2. $4y^{-1}$

3. $y = \dfrac{x+3}{5}$

4. $x \rightarrow \boxed{\times 6} \rightarrow \boxed{+9} \rightarrow z$

5. $3(x-4) = 2(8-2x)$
$3x - 12 = 16 - 4x$
$7x - 12 = 16$
$7x = 28$
$x = 4$

6. Complete the square:
$x^2 + 8x + 5 = (x+4)^2 - 16 + 5 = (x+4)^2 - 11$

7. $\dfrac{x^2 - 3x - 18}{x^2 + 4x + 3} = \dfrac{(x-6)(x+3)}{(x+1)(x+3)} = \dfrac{x-6}{x+1}$

8. $y = \dfrac{x+3}{x-2}$
$y(x-2) = x+3$
$xy - 2y = x+3$
$xy - x = 2y + 3$
$x(y-1) = 2y+3$
$x = \dfrac{2y+3}{y-1}$

9. $(3x-5)(x+4)$

10. $u_n = 3n^2 - 2n + 4$

11. Kasim has just added the numerators and added the denominators of the fractions. He should have written each fraction over a common denominator and then just added the numerators.
$\dfrac{2}{x+1} + \dfrac{1}{x-2} = \dfrac{2(x-2)+1(x+1)}{(x+1)(x-2)} = \dfrac{2x-4+x+1}{(x+1)(x-2)} = \dfrac{3x-3}{(x+1)(x-2)}$

12. $x^2 - 4x - 7 = 0$

$(x-2)^2 - 4 - 7 = 0$

$(x-2)^2 - 11 = 0$

$(x-2)^2 = 11$

$x - 2 = \pm\sqrt{11}$

$x = 2 + \sqrt{11}$ or $x = 2 - \sqrt{11}$

13. Let the two odd numbers be $(2x+1)$ and $(2x+3)$

$(2x+1)^2 = 4x^2 + 4x + 1$

$(2x+3)^2 = 4x^2 + 12x + 9$

Subtracting gives

$(2x+3)^2 - (2x+1)^2 = 4x^2 + 12x + 9 - (4x^2 + 4x + 1)$

$= 4x^2 + 12x + 9 - 4x^2 - 4x - 1$

$= 8x + 8$

$= 8(x+1)$

If x is an integer, $x+1$ is an integer so $8(x+1)$ is even.

14. Each term is 4 larger than the corresponding term in the first sequence so the formula for the nth term is $2n^2 - n + 9$

$2n^2 - n + 9 = 240$

$2n^2 - n - 231 = 0$

$(2n + 21)(n - 11) = 0$

$n = -10.5$ or $n = 11$

n can only be a positive integer so $n = 11$ and therefore 240 is a term in the sequence.

15. Try $x = 4.25$, $x^3 + x - 85 = -3.98...$

Try $x = 4.35$, $x^3 + x - 85 = 1.66...$

As there is a sign change between $x = 4.25$ and $x = 4.35$, 4.3 is a solution to one decimal place.

16. $2y - 3 > -5$

$2y > -2$

$y > -1$

$3y + 1 < 13$

$3y < 12$

$y < 4$

The integers which satisfy both inequalities are in the interval $-1 < y < 4$ so 0, 1, 2 and 3.

17. The first term is a; the second term is b; the third term is $a + b$; (the fourth term is $a + 2b$); the fifth term is $2a + 3b$.
Form two simultaneous equations:
$2a + 3b = 39 \quad \rightarrow 2a + 3b = 39$
$a + b = 11 \quad \times 3 \rightarrow 3a + 3b = 33$
Substituting gives $a = -6$
Substituting gives $b = 17$

18. Area of triangle $= \frac{1}{2}ab\sin C$

$$\frac{1}{2} \times x \times 6.8 \times \sin 30 = 14.28$$

$$x = \frac{14.28 \times 2}{6.8 \times \sin 30} = 8.4 \text{ cm}$$

19. Substitute $y = 2x - 1$ into $x^2 + y^2 = 9$

$$x^2 + (2x - 1)^2 = 9$$
$$x^2 + 4x^2 - 4x + 1 = 9$$
$$5x^2 - 4x - 8 = 0$$

Using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{4 \pm \sqrt{4^2 - 4 \times 5 \times -8}}{2 \times 5} = 1.7266... \text{ or } -0.9266\ldots \text{i.e. } 1.73 \text{ or } -0.927 \text{ (to 3 significant figures)}$$

Substitute into $y = 2x - 1$
If $x = 1.73$, $y = 2.46$
If $x = -0.927$, $y = -2.85$

So the coordinates of the points of intersection are (1.73, 2.45) and (–0.927, –2.85).

20. Let the width be x
$x(16 - 2x) \geq 24$
$-2x^2 + 16x - 24 \geq 0$
$(x - 2)(6 - x) \geq 0$

$2 \leq x \leq 6$ (in metres)

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GCSE (9–1)

MATHEMATICS

Section Check In

Assessment Objective	Qu.	Topic	R	A	G
AO1	1	Simplify an expression involving products of powers			
AO1	2	Simplify an expression involving quotients of powers			
AO1	3	Write an expression for an inverse function			
AO1	4	Complete a composite function			
AO1	5	Solve a linear equation involving brackets and unknowns on both sides of the equation			
AO1	6	Complete the square			
AO1	7	Factorise a quadratic expression to simplify an algebraic fraction			
AO1	8	Rearrange a formula where the subject appears twice			
AO1	9	Factorise a quadratic expression where $a \neq 0$ or 1			
AO1	10	Find a formula for the nth term of a quadratic sequence			
AO2	11	Add two algebraic fractions			
AO2	12	Solve a quadratic equation by completing the square			
AO2	13	Use algebra to construct a proof			
AO2	14	Use a formula for the nth term of a quadratic sequence			
AO2	15	Find an approximate solution to a cubic equation			
AO3	16	Solve two linear inequalities			
AO3	17	Set up and solve two simultaneous equations to solve a problem involving sequences			
AO3	18	Substitute values into a formula and solve to find an unknown			
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